Four operations on indices and surds, including rationalising the denominator

Indices and Surds

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Indices

e.g. 3^2, 5^4

base — 5^4 — index

plural: indices.

Surd - irrational number with roots,

e.g. $\sqrt{2}$, $\sqrt[3]{5}$, $\sqrt{2}+\sqrt{3}$, $3+\sqrt{5}$

But $\sqrt{4} = 2$ not surd $\because$ not irrational.

Laws of Indices

$$a^m \times a^n = a^{m+n}$$

$$(a^m)^n = a^{mn}$$

$$a^m \div a^n = a^{m-n}$$

$$a^0 = 1 \quad (0^0 \text{ not defined})$$

$$a^{-n} = \frac{1}{a^n}$$

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$$

$$a^m \times b^m = (a \times b)^m$$

$$a^m \div b^m = \left(\frac{a}{b}\right)^m$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

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e.g.

$$2^0 = 1$$

$$2^3 \times 2^4 = 2^{3+4}$$

$$(2^3)^4 = 2^{3\times 4}$$

$$2^{\frac{1}{3}} = \sqrt[3]{2}$$

$$2^{4/3} = \sqrt[3]{2^4} = (\sqrt[3]{2})^4$$

$$2^3 \times 4^3 = (2\times 4)^3$$

4 operations

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On surds

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

$$\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$$

$$m\sqrt{a} \pm n\sqrt{a} = (m \pm n)\sqrt{a}$$

$$m\sqrt{a} \pm m\sqrt{b} = m(\sqrt{a} \pm \sqrt{b})$$

e.g.

$$\sqrt{2} \times \sqrt{3} = \sqrt{2 \times 3}$$

$$\sqrt{2} \times \sqrt{3} = \sqrt{\frac{2}{3}}$$

$$6\sqrt{2} + 4\sqrt{2} = (6+4)\sqrt{2} = 10\sqrt{2}$$

$$6\sqrt{2} - 4\sqrt{2} = (6-4)\sqrt{2} = 2\sqrt{2}$$

$$4\sqrt{2} + 4\sqrt{3} = 4(\sqrt{2} + \sqrt{3})$$

$$4\sqrt{2} - 4\sqrt{3} = 4(\sqrt{2} - \sqrt{3})$$

Rationalise

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To change a surd in the denominator to a rational number.

e.g. $\frac{1}{\sqrt{2}}$. To rationalise:

$$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{\underbrace{2}_{\text{rational}}}$$

e.g. $\frac{1}{2+\sqrt{3}}$. To rationalise:

$$\frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$= \frac{2-\sqrt{3}}{2^2-3} = \frac{2-\sqrt{3}}{1}$$

$$= 2-\sqrt{3}$$

e.g. $\frac{1}{\sqrt{2}+\sqrt{3}}$. To rationalise:

$$\frac{1}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$$

$$= \frac{\sqrt{2}-\sqrt{3}}{2-3} = \frac{\sqrt{2}-\sqrt{3}}{-1}$$

$$= -\sqrt{2}+\sqrt{3}$$

Solving Equations

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2013 P2 Q8 (a) Find a, b for which $\dfrac{\sqrt{6}+\sqrt{5}}{\sqrt{15}-\sqrt{2}}$ can be expressed as $a\sqrt{10}+b\sqrt{3}$

Solution

$$\frac{\sqrt{6}+\sqrt{5}}{\sqrt{15}-\sqrt{2}} \times \frac{\sqrt{15}+\sqrt{2}}{\sqrt{15}+\sqrt{2}} = \frac{\sqrt{6\times15}+\sqrt{6\times2}+\sqrt{5\times15}+\sqrt{5\times2}}{15-2}$$

$$= \frac{3\sqrt{10}+2\sqrt{3}+5\sqrt{3}+\sqrt{10}}{13}$$

$$\boxed{\begin{aligned}\sqrt{6\times15} &= \sqrt{2\times3\times3\times5} = 3\sqrt{2\times5}\\ \sqrt{6\times2} &= \sqrt{2\times3\times2} = 2\sqrt{3}\\ \sqrt{5\times15} &= \sqrt{5\times3\times5} = 5\sqrt{3}\end{aligned}}$$

$$= \frac{4\sqrt{10}+7\sqrt{3}}{13}$$

$$= \underset{a}{\frac{4}{13}}\sqrt{10} + \underset{b}{\frac{7}{13}}\sqrt{3}$$

(b) Express $2^{2x} = 2^{2+x} + 21$ as a quadratic equation in 2^x and hence find x correct to 2 decimal places.

Let $X = 2^x$. So $2^{2x} = (2^x)^2 = X^2$

$$2^{2+x} = 2^2 \times 2^x = 2^2 X = 4X$$

Substituting, $X^2 = 4X + 21$

$X^2 - 4X - 21 = 0 \Rightarrow (X+3)(X-7) = 0$

$X = -3, 7$

$2^x = -3, 7$
reject

$x\log_{10}2 = \log_{10}7 \Rightarrow x = \dfrac{\log_{10}7}{\log_{10}2} =$ ___